Fairness in Streaming Submodular Maximization over a Matroid Constraint

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Monotone Submodular Function

Ground Set

$$f : 2^w \rightarrow R^+$$

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$$0 \leq f(e \mid X)$$

Any set X and element e

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Diminishing Returns

$$f(e \mid X \cup Y) \leq f(e \mid X)$$

Any sets X, Y and element e

Monotone Submodular Function

Influence Maximization

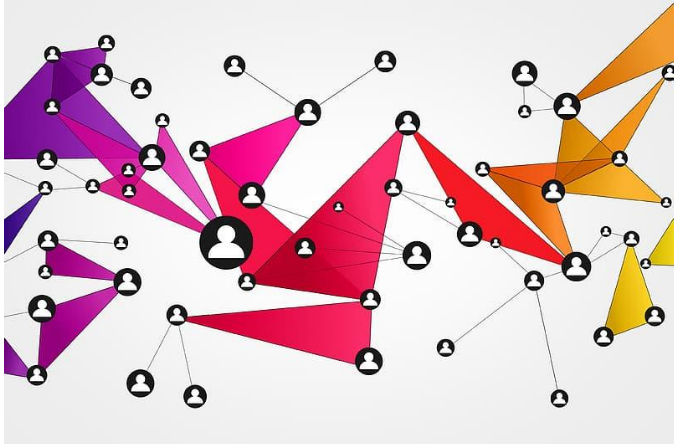


Image Summarization

Maximizing f under Cardinality Constraint

$$\max_{|S| \leq k} f(S)$$

Cardinality Constraint

Maximizing f under Matroid Constraint

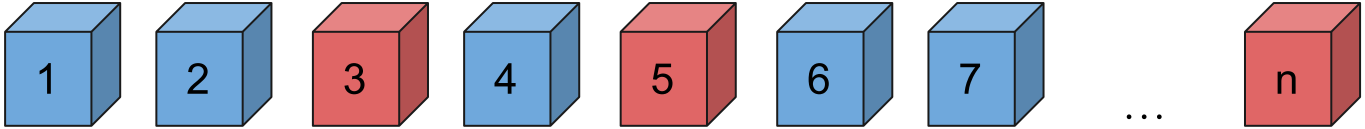
$$\max_{S \in \mathcal{M}} f(S)$$

Matroid Constraint

Fair Streaming Setting

- Elements arrive on a stream.
- We have limited memory.
- Each element has a color.
- We are given lower and upper bound constraint for each color.
    - The minimum and maximum number of elements that we can pick from each color.

Fair Streaming Setting



Find a solution such that

1. Number of blue elements in range [1, 2]
2. Number of red elements is in range [0, 3]
3. The solution belongs to a matroid

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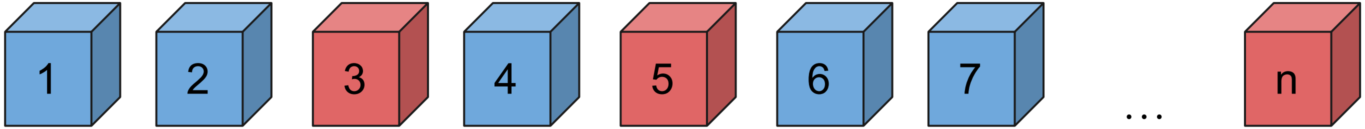

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The bounds are not constants

Our Results

A tight $\frac{1}{2}$-approximation algorithm with exponential memory

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Theorem 1.1. *For any constant* $\eta \in (0, 1/2)$, *there exists a one-pass streaming* $(1/2 - \eta)$*-approximation algorithm for FMMSM that uses* $2^{O(k^2 + k \log C)} \cdot \log \Delta$ *memory, where* $\Delta = \frac{\max_{e \in V} f(e)}{\min_{\{e \in V \mid f(e) > 0\}} f(e)}$.

Memory Usage

What if we want to use less memory?

Our Results

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Theorem 1.2. *Any (randomized) $o(\sqrt{\log C})$-pass streaming algorithm that determines the existence of a feasible solution for FMMSM with probability at least $2/3$ requires $\max(k, C)^{2-o(1)}$ memory.*

Our Results

If we violate the lower bounds we can get a high solution with quadratic memory usage in two passes over the stream

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Theorem 1.3. *There exists a two-pass streaming algorithm for FMMSM that runs in polynomial time, uses $O(k \cdot C)$ memory, and outputs a set S such that (i) S is independent, (ii) it holds that $\lfloor \ell_c/2 \rfloor \leq |V_c \cap S| \leq u_c$ for any color $c = 1, \ldots, C$, and (iii) $f(S) \geq \mathrm{OPT}/11.656$.*

Our Results

Even with more violations, it is not possible to get efficient algorithms.

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Theorem 1.4. *There is no one-pass semi-streaming algorithm that determines the existence of a feasible solution for FMMSM with probability at least* $2/3$*, even if it is allowed to violate the fairness lower bounds by a factor of* 2 *and completely ignore the fairness upper bounds.*

Overview of Our Algorithm

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Algorithm 1 FAIR-RESERVOIR

1: $I_c \leftarrow \emptyset$ for all $c = 1, ..., C$
2: **for** each element e on the stream **do**
3: Let c be the color of e
4: **If** $I_c + e \in \mathcal{I}$ **then** $I_c \leftarrow I_c + e$
5: Consider the partition matroid $\mathcal{I}_C$ on V defined in (1)
6: $S \leftarrow$ a max-cardinality subset of $\bigcup_c I_c$ in $\mathcal{I} \cap \mathcal{I}_C$ (Lemma 2.2)
7: **Return** S

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How can we do this?

→ Matroid intersection

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Algorithm 2 FAIR-STREAMING

Input: Set S from FAIR-RESERVOIR and routine $\mathcal{A}$
$S_1 \leftarrow \emptyset$, $S_2 \leftarrow \emptyset$
for e in S **do**
  Let c be the color of e
  if $|S_1 \cap V_c| < |S_2 \cap V_c|$ **then**
    $S_1 \leftarrow S_1 + e$
  else
    $S_2 \leftarrow S_2 + e$
Define matroids $\mathcal{I}^C, \mathcal{I}_1, \mathcal{I}_2$ as in Equations (2) and (3)
Run two copies of $\mathcal{A}$, one for matroids $\mathcal{I}^C, \mathcal{I}_1$ and one for matroids $\mathcal{I}^C, \mathcal{I}_2$, and let S'_1 and S'_2 be their outputs
for $i = 1, 2$ **do**
  for e in S_i **do**
    Let c be the color of e
    If $|S'_i \cap V_c| < u_c$ **then** $S'_i \leftarrow S'_i + e$
Return $S' = \arg\max(f(S'_1), f(S'_2))$

Open Directions

1. Other constraints
    - Knapsack constraint

2. Single pass algorithm with efficient memory

3. Stronger impossibility results