A Constant-Factor Approximation Algorithm for the Asymmetric Traveling Salesman Problem

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Subject: Held-Karp LP relaxation

Variables: \( x_{uv} \) = times we traverse edge \((u, v)\)

Minimize: \( \sum_{e \in E} w(e) x_e \)

Subject to: \( x(e^+)^S = x(e^-)^S \) for all \( e \in E \) (for a cycle \( S \))

where \( x(e) \) = 1 or 0

What is the integrity gap?

Local-Connectivity ATSP

[Svensson’15]:
- Defined new, easier problem called Local-Connectivity ATSP
- Reduced \( O(1) \)-apx of ATSP to Local-Connectivity ATSP
- Solved Local-Connectivity ATSP for unweighted graphs (easy part)\footnote{Svensson, 2015}

Thus: \( O(1) \)-apx of ATSP for unweighted graphs

Then we [16] solved Local-Connectivity ATSP for graphs with two different edge weights. But could not generalize even to three weights...

Make the instance laminarily-weighted

Solve dual, uncross, use complementary slackness, and rewrite objective function to get the following structure:

\[ w(e) = \sum_{S \in \mathcal{L}} y_S \]

(\( \mathcal{L} \) = laminar family of cuts)

- Primal LP value:
  \[ \sum_{e \in E} w(e) x_e = 2 \cdot \sum_{S \in \mathcal{L}} y_S \]

Irreducible \( \Rightarrow \) close to Hamiltonian

If all sets are close to Hamiltonian, then we have enough structure to solve Local-Connectivity ATSP using previous work, circulations, ...

Future work

- Our \( \alpha \)p ratio is not close to 2. At all. Need new ideas to get much better approximation algorithms for ATSP.
- Bottleneck ATSP problem: find tour (visit each vertex exactly once) with minimum max edge weight. Beat \( O(\log n / \log \log n) \)-apx.
- Thin tree conjecture: for every \( K \), there is \( O(1) \)-apx of \( K \)-approximation for Bottleneck ATSP
- Node-weighted symmetric TSP, i.e. \( w(u, v) = f(u) + f(v) \): beat 1.5-apx.

Strategy: contract, recurse, lift, complete

- Set dual value for new singleton to support payment for patching up when lifting the tour later, i.e. set it to max possible length of a red path below
- LP-value = \( OPT - drop \)
- \( S \) is reducible \( \Rightarrow \) large drop

Return an approximate solution: \( w(\text{tour}) \leq a \cdot OPT \)